3 **Course Content**

**3.1. Numbers and expressions**

3.1.1. Real numbers: natural, integer, rational, irrational. Divisibility rules. The absolute value of a real number, the properties of absolute value, the geometric meaning of |a| and |a - b|. The solution of the simplest equations and inequalities containing the symbol of the absolute value.

3.1.2. Percentages and proportions.

3.1.3. Powers and roots. Rules of action on them.

3.1.4. Transformations of numerical and algebraic expressions.

**3.2. Properties of the simplest functions**

3.2.1 Definition of a function, domain and codomain of a function, even and odd functions. Graph of a function. Graphs transformations.

3.2.2. Linear function $y = kx + b$. Geometric meaning of $k$ and $b$. Function $y = \frac{k}{x}.$

Fractional - linear function.

3.2.3. Quadratic function. Completing the square. Plotting the simplest functions and functions that contain the argument under the absolute value symbol. Conversion and Plotting

**3.3. Equations and systems of equations**

3.3.1. Equations. General provisions. The notion of equivalence of equations.

3.3.2. Linear equations, systems of linear equations. Geometric interpretation of systems of linear equations. Equations and systems that can be reduced to linear. Solving examples and text problems.

3.3.3. The quadratic equations and the equations that are reducible to them.

3.3.4. Vieta’s theorem and its application to problem solving.

3.3.5. Polynomials with one variable. Finding rational and integer roots of a polynomial with integer coefficients. Polynomial division. The Bezout’s theorem.

3.3.6. The solution of equations of higher powers.

3.3.7. The solution of equations containing the symbol of the absolute value. Plotting functions containing an absolute value symbol.

3.3.8. Irrational equations. The emergence of "extra" roots. Raising both sides of the equation to an even degree.

3.3.9. Solving algebraic systems of equations

3**.4. Inequalities**

3.4.1. Properties of numerical inequalities. Actions over inequalities.

3.4.2. The solution of linear, square inequalities and inequalities that can be reduced to them.

3.4.3. Geometric interpretation of the solution of systems of linear inequalities.

3.4.4. Rational inequalities. Features of working with inequalities. Scheme of solutions of rational inequalities. Interval method.

3.4.5. Solution of inequalities containing symbol of an absolute value.

3.4.6. Irrational inequalities. Equivalent transformations.

**3.5. Trigonometry**

3.5.1. Trigonometric circle. Definition of trigonometric functions. Properties of trigonometric functions, graphs.

3.5.2. Basic trigonometric formulas. Identical transformations of trigonometric expressions

3.5.3. Trigonometric equations and inequalities. The main methods for solving equations. Selection of solutions. The solution of trigonometric inequalities using a trigonometric circle.

3.5.4. Systems of trigonometric equations.

3.5.5. Trigonometric equations and inequalities with a parameter. Solution of trigonometric inequalities

**3.6. Tasks for compiling equations**

3.6.1. Tasks for movement.

3.6.2. Tasks for work

3.6.3. Tasks for percent.

3.6.4. Tasks for mixtures and alloys

3.6.5. Tasks with integer unknown quantitites.

3.6.6. Arithmetic progression, geometric progression and infinitely decreasing geometric progression. Solving examples and tasks.

**3.7. Logarithmic and exponential equations and inequalities**

3.7.1. The solution of the simplest exponential equations. Logarithm of a number. Basic logarithmic identity. Actions on logarithms. Taking the logarithm and the exponent of a number. Transition module. Identical transformations of logarithmic and exponential expressions

3.7.2. Exponential and logarithmic functions. Properties and their graphics.

3.7.3. The main types of exponential equations and inequalities and methods for their solution.

3.7.4. The main types of logarithmic equations and inequalities and methods for their solution

3.7.5. Non-algebraic systems of equations

3.7.6. Basic methods for solving logarithmic equations and inequalities

**3.8. Derivative and its application**

3.8.1. Calculation of the derivative, the rules of differentiation.

3.8.2. The geometric meaning of the derivative. Equation of a tangent to the graph of the function.

3.8.3. Investigation of functions using a derivative.

**3.9. Planimetrics**

3.9.1. Triangles.

3.9.2. Parallelograms and trapeziums.

3.9.3. Circles.

**3.10. Solid geometry**

3.10.1. Polyhedra

3.10.2. Polyhedra Sections

3.10.3. Solid of revolution

3.10.4. Combinations of polyhedra and Solids of revolution

3.10.5. Construction of polyhedron cross sections

**3.11. Tasks with parameters**

3.11.1. The existence of roots of a quadratic equation. Signs of the roots.

3.11.2. The location of the roots of the square trinomial, depending on the parameter.

3.11.3. Equations, inequalities and systems of equations with parameters. Graphic interpretations.

Basic methods for solving problems with parameters

**3.3. Examination card example**

Ministry of Education and Science of the Russian Federation

National Research Tomsk State University

MATHS

Option 19

Instructions for performing the examination paper

To complete the exam in mathematics is given 4 hours. The exam consists of two parts and contains 16 tasks.

Task conditions are not overwritten. All tasks are necessarily performed in draft. In the absence of solving problems in a draft, these tasks are not counted. In a fair copy, completing each task, write down a complete informed decision and answer. Answers to the tasks of Part I in a fair copy should be entered in the answer table, constructed according to the following example.

Table for answers to the tasks of part I

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Task number** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **Total** |
| **Answer** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **Points** |  |  |  |  |  |  |  |  |  |  |  |  |  |

Part 1

1. A student bought a month pass and made 48 trips in a month. How many rubles did he save, if a month pass costs 500 rubles, and a one-time trip costs 24 rubles?

(3 points)

2. The price of the electric kettle was increased by 21% up to 1,815 rubles. How much did the kettle cost before the price increase?

(3 points)

3. Calculate (not using tables or calculator): $(\frac{11}{15}-1\frac{9}{10}+\frac{5}{18})\*0.9+0.1$

(4 points)

4. Calculate $cos(alpha)$, if $tg\left(alpha\right)=0,75, π<alpha<\frac{3π}{2}$

(4 points)

5. Solve the equation $\frac{3x}{2-x}=\frac{4}{x}-3$

(4 points)

6. Solve the equation $(1/2)^{3x-12}=8$

(4 points)

7. Solve the equation $\sqrt{16-6x}=x$. If the equation has more than 1 root, specify the least root in your answer.

(5 points)

8. Solve the inequality $x\leq -\frac{4x}{2-x}$. In your answer specify the amount of positive integer solutions.

(5 points)

9. In the triangle ABC, where $AC=BC=20, AB=32$ find the sine of A angle.

(5 points)

10. In a cylindrical vessel the liquid level reaches 8 cm. At what height will the liquid level be, if it is poured into a second cylindrical vessel whose diameter is three times smaller than the diameter of the first? The answer is expressed in centimeters.

(6 points)

11. Three teams produced together 114 spare parts. It is known that the second team has manufactured the spare parts 3 times more than the first and 16 less parts than the third. How many parts more produced the third team than the first.

(6 points)

12. Rectilinear motion of a point is described by the law $S(t) = —t^{3}-6t^{2} + 20t$. (Time

measured in seconds, movement - in meters.) Find the lowest speed of its movement.

(6 points)

Part 2

13. Solve the equation $6cos^{2}\left(x\right)+5\sin(\left(x\right))-2=0$ and specify roots within the range $[-2π,-\frac{π}{2}]$

(10 points)

14. Solve the inequality $log\_{5-x}(x^{2}-2x+1)\leq 2$

(10 points)

15. First circle is inscribed in a rectangular trapezoid, and the second touches a larger side and continuations of the bases. Find the distance from the top of one of the right angles of the trapezoid to the center of the second circle if the point of tangency of the first circle with the larger side of the trapezoid divides it into segments equal to 2 and 8.

(10 points)

16. Find all the values of the parameter $a$, for each of which the set of solutions to the inequality

$$\frac{a-\left(a^{2}-2a+0.5\right)\sin(\left(x\right))+4}{(\cos(\left(x\right)))^{2}+a^{2}+1}<1$$

Contains interval $[0,\frac{5π}{6}]$

(15 points)